

DEBYE SCREENING, KMS AND IMAGINARY-TIME FORMALISM OF TEMPORAL AXIAL GAUGE ¹

S.M.H. WONG

²*LPTHE, Université de Paris XI, Bâtiment 211, F-91405 Orsay, France*

We argue that in QCD, the Debye mass requires not only a mathematical definition but also a physical one and temporal axial gauge could provide for a physical screening potential for this purpose. Unfortunately, this gauge is spoiled by problem of energy conservation rather than the well known divergence due to the double pole in the longitudinal propagator. We also show that KMS condition is violated in this gauge and is therefore not universally true.

1 Why Imaginary-Time and Why Temporal Axial Gauge?

Since the beginning of the nineties, it is known that at finite temperature, in order to do true perturbative calculations order by order in the coupling constant, it is necessary to perform Braaten-Pisarski resummation[1] and use effective propagators and vertices in place of the bare quantities. This resummation requires the hard thermal loops[2] of the N-point functions. These are essentially the leading terms of the one-loop N-point functions. In real-time, there is the doubling of the degrees of freedom, so in principle, one will have to work out all the components of each N-point hard thermal loops before one can do resummation. This is, however, not necessary in imaginary-time. There is only one hard thermal loop for each N-point function so imaginary-time is comparatively simple to work in. Furthermore, when considering static problem like calculating the Debye mass at the next to leading order,

¹Talk presented at Thermo95, Dalian, China

²Laboratoire associé au Centre National de la Recherche Scientifique

there is the simplification of it is only necessary to keep the zero mode [3, 4] in the Matsubara frequency sum. Other modes are irrelevant at this order.

Temporal axial gauge (TAG) offers one the chance to work in an “abelianized” gauge field theory as long as one restricts oneself to the static chromo-electric sector. Therefore in this gauge, one can say with certainty, that the gauge invariant static quark-antiquark potential in a hot medium is directly related to the longitudinal gluon propagator alone. In other gauges, 3-point and/or 4-point function cannot be easily excluded. This will be shown more explicitly in Sec. 2. TAG has another feature which is the absence of ghost, that means one cannot include the whole Hilbert space in the partition function which leads to the violation of KMS boundary condition. This will be shown in Sec. 3.

Having mentioned the physical reasons and advantages of working in TAG, one must not forget the disadvantage. It is well known that the longitudinal propagator has a troublesome double pole $1/k_0^2$ at $T=0$. To handle this pole, some prescription is required to displace it away from the real energy axis. At finite temperature, if one simply turns the $T=0$ propagator into the finite T propagator by giving it discrete imaginary energy, one will be facing immediately a divergence at zero energy. This problem has traditionally been dealt with by simply dropping the infinity[5]. It is found to be correct at leading order but at higher order it is almost certainly not correct. It is simple to understand why dropping the divergence will not affect the leading order result. Since the leading terms are essentially the hard thermal loops which do not get any contribution from the soft zero mode and the divergence is precisely coming from this mode. In the following, it will be shown that in fact this problem of the double pole does not exist at finite T in the imaginary-time formalism, however, the hope of using a trouble free imaginary-time formalism of TAG to study physical problems can still not be fulfilled.

2 Debye Screening

2.1 Debye Screening in TAG

The potential of a charge Q_1 at \mathbf{x}_1 in the presence of another charge Q_2 at \mathbf{x}_2 is given by [6]

$$V(\mathbf{r}) = \frac{1}{2} \sum_a \int d^3\mathbf{x} \left(\mathcal{E}_1^{a \text{ eff}}(\mathbf{x}) \cdot \mathcal{E}_2^a(\mathbf{x}) + \mathcal{E}_2^{a \text{ eff}}(\mathbf{x}) \cdot \mathcal{E}_1^a(\mathbf{x}) \right) \quad (1)$$

where the effective field $\mathcal{E}_1^{a \text{ eff}}$ created by the non-abelian charge Q_1^a is the sum of the applied field \mathcal{E}_1^a and the induced field $\delta\langle \mathbf{E}_1^a \rangle$:

$$\mathcal{E}_1^{a \text{ eff}}(\mathbf{x}) = \mathcal{E}_1^a(\mathbf{x}) + \delta\langle \mathbf{E}_1^a \rangle \quad (2)$$

\mathcal{E}_1^a is a solution of the Gauss' law:

$$\nabla \cdot \mathcal{E}^a - g f^{abc} \mathcal{E}^b \cdot \mathcal{A}^c = Q_1^a \delta^3(\mathbf{x} - \mathbf{x}_1) , \quad (3)$$

where \mathcal{A} is the vector potential associated with \mathcal{E} . The potential Eq. (1) is manifestly gauge invariant and is therefore physical.

In TAG, the electric field is linear in the vector potential

$$E_i^a(\mathbf{x}, t) = -\partial_0 A_i^a(\mathbf{x}, t) , \quad (4)$$

and in a static situation, A 's depend linearly in time so Gauss' law becomes abelian. It is now simple to solve and the solution is

$$\mathcal{E}_i^a(\mathbf{x}) = -iQ_1^a \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}_1)} \frac{k_i}{k^2} . \quad (5)$$

We stress that in gauges such as covariant or Coulomb gauge, Gauss' law is not abelian and is therefore not trivial to solve[4]. The form Eq. (5) plays an important role in determining to which N-point function the physical screening potential Eq. (1) is related.

The coupling Hamiltonian which couples the external applied field \mathcal{E}_1^a to the field in the medium \mathbf{E}_1^a is

$$H^{\text{ext}}(t) = \int d^3\mathbf{x} \mathbf{E}_1^a(\mathbf{x}, t) \cdot \mathcal{E}_1^a(\mathbf{x}) . \quad (6)$$

So from linear response theory, the induced field is

$$\begin{aligned} \langle E_i^a(\mathbf{x}, t) \rangle &= i \int_{-\infty}^t dt' \langle [H^{\text{ext}}(t'), E_i^a(\mathbf{x}, t)] \rangle \\ &= i \int_{-\infty}^t dt' \int d^3\mathbf{x}' \mathcal{E}_j^b(\mathbf{x}') \langle [E_j^b(\mathbf{x}', t'), E_i^a(\mathbf{x}, t)] \rangle . \end{aligned} \quad (7)$$

The correlator $\langle [E, E] \rangle$ can be written in terms of the retarded gluon propagator because of Eq. (4), so the effective field in the plasma is

$$\mathcal{E}_i^{a \text{ eff}}(\mathbf{x}) = \int \frac{d^3 \mathbf{k}}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{x}} \mathcal{E}_j^b(\mathbf{k}) \lim_{\omega \rightarrow 0} [\omega^2 D_{ij}^{ab R}(\omega, \mathbf{k})] . \quad (8)$$

After putting everything back into Eq. (1), we see that the retarded propagator is now contracted between two \mathcal{E} 's which project out the longitudinal component of the propagator. So in TAG, the screening potential is directly related only to the longitudinal gluon propagator. If Eq. (5) is not of such a form, then this would not be true which is the case in other gauges.

2.2 Definition of the Debye Mass

In this section, we would like to argue that although the new definition of the Debye Mass proposed by Rebhan defined at the pole of the longitudinal propagator

$$m^2 = \lim_{k^2 \rightarrow -m^2} \Pi_{00}(0, k) \quad (9)$$

is significantly improved over the old definition

$$m^2 = \lim_{k^2 \rightarrow 0} \Pi_{00}(0, k) \quad (10)$$

in the sense that it is self-consistent and also gauge[7] and renormalization group invariant[8]. It still remains only a good mathematical definition. It is necessary but not sufficient. Because to be certain that this mass is indeed the Debye mass, one will have to first find a physical static quark-antiquark potential which has an exponential form at large spatial separation and then the inverse screening length will have to be exactly given by Eq. (9).

As explained in the previous section, it is not clear that one can look for screening behaviour using only the 2-point function in gauges other than TAG. For example, in covariant and Coulomb gauge, the screening function[3] based on the longitudinal gluon propagator is not gauge invariant and is therefore not physical. So even if it behaves exponentially at large distance, one still cannot say that the screening length of this function is the inverse of the Debye mass. This is further complicated by the need to introduce by hand the magnetic mass both to remove infrared divergence and to ensure gauge invariance[7]. For a discussion on the consistency of this, we refer to the paper of Blaizot and Iancu[9].

3 KMS is not Universal

From quantum mechanics, the probability amplitude for evolving from a state q at time t to a state q' at time t' can be written as

$$\langle q' t' | q t \rangle = \int [dq] \exp\{-i \int_t^{t'} dt L\} . \quad (11)$$

If one applies this to the partition function for, say scalar field theory,

$$\begin{aligned} Z_\phi &= \sum_\phi \langle \phi | \exp(-\beta H_\phi) | \phi \rangle \\ &= \int_{\text{periodic}} [d\phi] \exp\{-i \int_0^{-i\beta} dt L_\phi\} . \end{aligned} \quad (12)$$

The interpretation is that one starts from a state ϕ at $t = 0$ and evolves back through a time $-i\beta$ to ϕ , so ϕ has to be periodic in $-i\beta$. In the case of free gauge fields in TAG, since only transverse (physical) states are included in the thermal average, the partition function is

$$\begin{aligned} Z_A &= \sum_T \langle T | \exp(-\beta H_A) | T \rangle \\ &= \int_{\text{periodic}} [dA_T] \exp\{-i \int_0^{-i\beta} dt L_{A_T}\} . \end{aligned} \quad (13)$$

The longitudinal field part of the Hamiltonian has only the vacuum to act on so there is no path integral for the longitudinal field. We see that the transverse field must be periodic in time but the longitudinal field is not required to be so.

Periodic field implies KMS boundary condition therefore the transverse propagator is periodic. Whereas the longitudinal field is not periodic, moreover, there is no trace identity $\text{tr}(AB) = \text{tr}(BA)$ due to the unphysical part of the Hilbert space is excluded so the longitudinal propagator does not satisfy KMS. This feature is also true in real-time. Therefore KMS does not hold universally as is widely assumed.

4 The Double Pole $1/k_0^2$ Problem does not exist at Finite T

We start by setting $A_0 = 0$ which we can do if there is no divergence due to the $1/k_0^2$. We will assume this and check that this is the case below.

Continuing to work in the free field case, the lagrangian is

$$\mathcal{L} = \frac{1}{2}(\dot{\mathbf{A}}^2 + \mathbf{A}_T \partial^2 \mathbf{A}_T) \quad (14)$$

so the equation of motion of the longitudinal field is $\ddot{\mathbf{A}}_L = 0$ and it is not a wave equation.

In order to quantize \mathbf{A}_L , we write down a general form for it

$$\mathbf{A}_L(\mathbf{x}) = \int \frac{d^3 \mathbf{k}}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{x}} [\alpha(\mathbf{k}) + \beta(\mathbf{k}) t] \hat{\mathbf{k}}/k . \quad (15)$$

\mathbf{A}_L is Hermitian so we can rewrite this as

$$\mathbf{A}_L(\mathbf{x}) = \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \left\{ e^{i\mathbf{k} \cdot \mathbf{x}} [\alpha(\mathbf{k}) + \beta(\mathbf{k}) t] + e^{-i\mathbf{k} \cdot \mathbf{x}} [\alpha^\dagger(\mathbf{k}) + \beta^\dagger(\mathbf{k}) t] \right\} \theta(k_3) \hat{\mathbf{k}}/k . \quad (16)$$

k_3 in the theta function is of course arbitrary. One can equally choose k_1 or k_2 . The commutation relations for the operators, α , β , α^\dagger and β^\dagger are to be fixed by the canonical commutation relations.

Because of the presence of the theta function, canonical commutation relations such as $[\mathbf{A}_L(\mathbf{x}), \mathbf{A}_L(\mathbf{y})] = 0$ and $[\dot{\mathbf{A}}_L(\mathbf{x}), \dot{\mathbf{A}}_L(\mathbf{y})] = 0$ cannot be satisfied trivially with the usual type of commutation relations like $[\alpha(\mathbf{k}), \alpha^\dagger(\mathbf{k}')] = 2k (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}')$ and a similar one for β and β^\dagger . Instead, one is obliged to choose

$$\begin{aligned} [\alpha(\mathbf{k}), \alpha^\dagger(\mathbf{k}')] &= 0 , & [\beta(\mathbf{k}), \beta^\dagger(\mathbf{k}')] &= 0 , \\ [\alpha(\mathbf{k}), \beta^\dagger(\mathbf{k}')] &= i (2\pi)^3 \mathbf{k}^2 \delta(\mathbf{k} - \mathbf{k}') \end{aligned} \quad (17)$$

and the Hermitian conjugate of the last relation above.

With Eq. (16) and Eq. (17), we can now work out the longitudinal gluon propagator in configuration space. The momentum space representation can be obtained by Fourier transform and the full time range, i.e. from $-i\beta$ to $i\beta$, must be used in order to go to energy space due to the lack of periodicity. Since the physical states are the $|T\rangle$'s so the longitudinal gluon propagator is a $T=0$ propagator. It is not heated at the lowest order and it has the form[10, 11]

$$D_{ij}^L(t, \mathbf{k}) = \frac{k_i k_j}{\mathbf{k}^2} T \sum_{k_0 \text{ odd, even}} D_{k_0}^L(\mathbf{k}) e^{ik_0 t} . \quad (18)$$

The momentum space form of the zero mode component is $D_{k_0=0}^L(\mathbf{k}) = 1/4T^2$. So we see that the double pole $1/k_0^2$, in fact, does not exist in agreement with the assumption we made at the beginning of this section.

5 Energy Conservation

In the last section, we see that D^L has both even and odd modes which are commonly named erroneously bosonic and fermionic modes respectively. The presence of both types of mode in a propagator is actually problematic, if one recalls how energy conservation is ensured in the usual case. In any interaction, it is the time integration at each vertex which gives the important energy conserving delta function. Consider any 3-point interaction in imaginary-time, the time integration is of the form

$$\int_0^{-i\beta} dt e^{i(k_{01}+k_{02}+k_{03})t} = -i\beta \delta_{k_{01}+k_{02}+k_{03},0} \quad (19)$$

provided the sum of the energies $k_{0i}, i = 1, 2, 3$ of the three incoming particles is an even multiple of $2\pi iT$. This is always the case in non-gauge theories. In a gauge theory, eg. QED in TAG, this is not always the case because the sum of the energies of the interaction $e\bar{\psi}\gamma^i\psi A_{Li}$ can now be both even and odd. So the energy conservation mechanism that one usually has is broken in this case[11].

If one looks at this from another angle, one can write down a thermal N-point function in terms of thermal average over physical states of N Heisenberg fields. Re-expressing everything in the interaction picture and using the properties of the interaction picture fields to introduce a time shift, say δ , to every one of the N fields. The resulting expression[11] differs from the initial expression in the interaction picture by only the time shift δ in the N fields plus $\exp(\pm i\beta H)$ on either side of the kernel of the matrix elements, acting on the enclosing $\langle phys|$ and $|phys\rangle$. Since physical states can be rewritten as energy eigenstates of H , so the exponential operators become c-numbers and cancel each other. Therefore any N-point function satisfies

$$\Gamma^N(t_1, t_2, \dots, t_N) = \Gamma^N(t_1 + \delta, t_2 + \delta, \dots, t_N + \delta) \quad (20)$$

and since time-translation invariance implies energy conservation so the latter still somehow seems to hold, despite the fact that the simplest energy conservation mechanism is broken.

6 Indefinite Metric Field Theory

In choosing TAG and setting $A_0 = 0$, \mathbf{A}_L depends linearly on time and on four operators which satisfy Eq. (17). Using these, one can construct states of the form

$$|\psi\rangle = \prod_{\mathbf{k}} (\alpha^\dagger(\mathbf{k}))^{m_{\mathbf{k}}} (\beta^\dagger(\mathbf{k}))^{n_{\mathbf{k}}} |0\rangle, \quad (21)$$

where $m_{\mathbf{k}}$ and $n_{\mathbf{k}}$ are both integers. The simplest states are

$$|\alpha\rangle = \alpha^\dagger(\mathbf{k})|0\rangle, \quad (22)$$

$$|\beta\rangle = \beta^\dagger(\mathbf{k})|0\rangle, \quad (23)$$

$$|\alpha, \beta\rangle = \alpha^\dagger(\mathbf{k})\beta^\dagger(\mathbf{k})|0\rangle, \quad (24)$$

of which, the first two have zero norms. While the second is an eigenstate of the free longitudinal Hamiltonian H_0^L of zero eigenvalue, the other two are not. Furthermore, they cannot be made into eigenstates of H_0^L by superposition. The Hilbert space of H_0^L is in fact spanned by an infinite number of null and non-null states given by Eq. (21), not all of which are eigenstates of H_0^L . In the language of indefinite metric field theory[12], this Hilbert space is spanned instead by the generalized eigenstates of H_0^L .

The generalized eigenstates $|\omega\rangle$ of a Hamiltonian \mathcal{H} and their corresponding generalized eigenvalues ω are defined by

$$\begin{aligned} (\mathcal{H} - \omega)^p |\omega\rangle &= 0 && \text{for } p \geq n, \\ &\neq 0 && \text{otherwise,} \end{aligned} \quad (25)$$

for some integer n . In our present case, the generalized eigenvalues are all zeros and the n for the first and third states above is 2.

Generalized eigenstates are of course not the same as eigenstates if $n \neq 1$, so the argument at the end of Sec. 5 unfortunately does not work in the final step. So although the problem of the double pole no longer exists, we are facing a new obstacle of energy conservation forced upon by the Hamiltonian formulation.

Acknowledgments

The author thanks the organizers and the people of Dalian University of Technology for all their efforts and also H. Matsumoto for helpful discussions.

The author acknowledges financial support from the Leverhulme Trust.

References

References

- [1] E. Braaten and R.D. Pisarski, *Nucl. Phys. B* **337**, 569 (1990).
- [2] J. Frenkel and J.C. Taylor, *Nucl. Phys. B* **334**, 199 (1990).
- [3] A.K. Rebhan, *Phys. Rev. D* **48**, R3967 (1993).
- [4] S. Peigné and S.M.H. Wong, *Phys. Lett. B* **346**, 332 (1995).
- [5] U. Heinz, K. Kajantie and T. Toimela, *Ann. Phys.* **176**, 218 (1987).
- [6] K. Kajantie and J. Kapusta, *Ann. Phys.* **160**, 477 (1985).
- [7] R. Kobes, G. Kunstatter and A.K. Rebhan, *Phys. Rev. Lett.* **64**, 2992 (1990).
- [8] A.K. Rebhan in Proceedings of the 3rd Workshop on Thermal Field Theories and Their Applications, Banff, Canada, August 1993, World Scientific, Edited by F.C. Khanna, R.Kobes, G. Kunstatter and H. Umezawa, p.469.
- [9] J.P. Blaizot and E. Iancu, preprint Saclay-T95/055, IPN-Orsay/TH 95-28, hep-ph/9506438, 1995.
- [10] K. James and P.V. Landshoff, *Phys. Lett. B* **251**, 167 (1989).
- [11] S.M.H. Wong, preprint LPTHE-Orsay 94/114, to appear in Zeitschrift für Physik C, hep-th/9501032.
- [12] N. Nakanishi, *Prog. Theor. Phys. Suppl.* **51**, 1 (1972).